

MATH4050 Real Analysis

~~Assignment 4~~ HW 4 - 2022

There are 5 questions in this assignment. The page number and question number for each question correspond to that in Royden's Real Analysis, 3rd or 4th edition.

1. (3rd: P.64, Q9)

Show that if E is a measurable set, then each translate $E + y$ of E is also measurable.

2. (3rd: P.64, Q10)

Show that if E_1 and E_2 are measurable, then

$$m(E_1 \cup E_2) + m(E_1 \cap E_2) = mE_1 + mE_2.$$

3. (3rd: P.64, Q11)

Show that the condition $mE_1 < \infty$ is necessary in Proposition 14 (3rd edition) by giving a decreasing sequence $\{E_n\}$ of measurable sets with $\phi = \bigcap E_n$ and $mE_n = \infty$ for each n .

★ 4. (3rd: P.70, Q21)

- Let D and E be measurable sets and f a function with domain $D \cup E$. Show that f is measurable if and only if its restrictions to D and E are measurable.
- Let f be a function with measurable domain D . Show that f is measurable iff the function g defined by $g(x) = f(x)$ for $x \in D$ and $g(x) = 0$ for $x \notin D$ is measurable.

✂ 5. (3rd: P.71, Q22)

- Let f be an extended real-valued function with measurable domain D , and let $D_1 = \{x : f(x) = \infty\}$, $D_2 = \{x : f(x) = -\infty\}$. Then f is measurable if and only if D_1 and D_2 are measurable and the restriction of f to $D \setminus (D_1 \cup D_2)$ is measurable.
- Prove that the product of two measurable extended real-valued functions is measurable.
- If f and g are measurable extended real-valued functions and α a fixed number, then $f + g$ is measurable if we define $f + g$ to be α whenever it is of the form $\infty - \infty$ or $-\infty + \infty$.
- Let f and g be measurable extended real-valued functions that are finite almost everywhere. Then $f + g$ is measurable no matter how it is defined at points where it has the form $\infty - \infty$.